ROAD
algorithm for control charts

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Outline:

1. Introduction
2. Robust control charts
3. Adaptive control charts
4. ROAD control chart
5. Conclusions

1. Introduction

The most commonly used control charts (CC) for monitoring industrial processes are control charts of Shewhart type

Shewhart CC with control limits $\mu_0 \pm 3\sigma_0/\sqrt{n}$ works well only under the assumptions of independent and normally distributed data

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- outliers
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- periodicity, seasonal disturbances
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frequent false alarms!

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Average Run Length (ARL):

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θ - controlled parameter
$\pi_W(\theta)$ - power function of the W-chart

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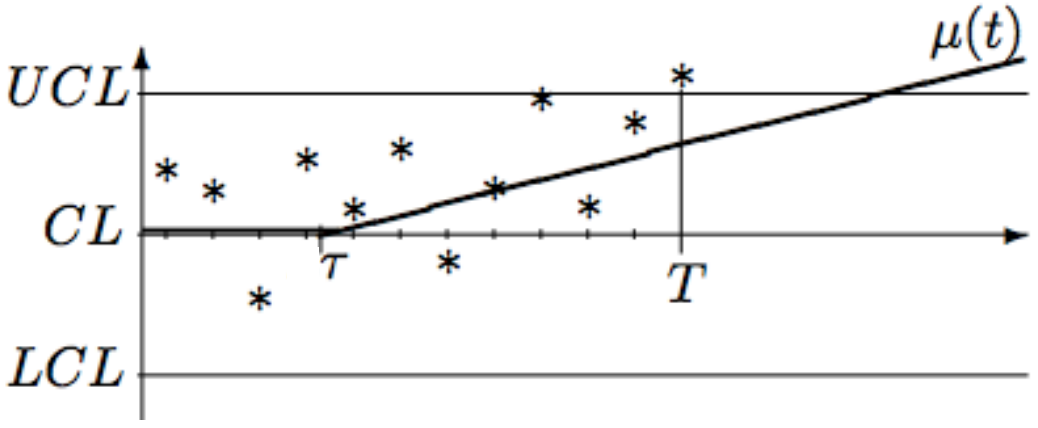
Average Delay (ADEL)

$$ADEL = \frac{1}{\delta} E\big(T - \tau | T > \tau\big)$$



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dependence of observations => use of historical observations

- Montgomery, D.C. (2005) Introduction to Statistical Quality Control, Wiley, New York
- Reynolds, M.R., Stoumbos, Z.G. (2010) Robust CUSUM charts for monitoring the process mean and variance, Quality and Reliability Engineering International **26**, 453-473
- Lee, H.CH., Apley, D.W. (2011) Improved design of robust exponentially weighted moving average control charts for autocorrelated processes, Quality and Reliability Engineering International **27**, 337-352

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- Chakraborti, S., Van der Laan, P., Van de Wiel, M.A. (2001) Nonparametric Control Charts: An Overview and Some Results. *Journal of Quality Technology* **33**, 304-315
- Bakir, S.T. (2006) Distribution-Free Quality Control Charts Based on Signed Rank Like Statistics. *Communications in Statistics, Theory and methods*, **35**, 734-757

2. Robust control charts

1) Shewhart type $\tilde{X}$ control chart

$$LCL = -3.025,$$
$$UCL = 3.025$$

2) EWMA $\tilde{X}$ control chart

$$\tilde{Z}_{n+1} = \gamma \tilde{X}_n + (1-\gamma)\tilde{Z}_n$$
$$\gamma = 0.1, \quad L = 2.827$$

3) CUSUM $\tilde{X}$

$$\tilde{C}^+_{n+1} = \max\left[0, \tilde{C}^+_n - (\mu_0 + \delta_0) + \tilde{X}_n\right]$$
$$\tilde{C}^-_{n+1} = \max\left[0, \tilde{C}^-_n + (\mu_0 + \delta_0) - \tilde{X}_n\right]$$
$$\delta_0 = 0.15, \quad L = 4.344$$

3. Adaptive control charts

In control chart design and implementation, there are two sets of parameters required to be determined:

- sampling parameters (sample size, sampling interval)
- design parameters (limits, parameters of charting statistics)

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Alternative is to improve estimation accuracy by using samples collected in Phase II => adaptive control charts

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easily realized as long as SPC schemes are implemented with the aid of computers

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3. Adaptive control charts

Examples of charts with adaptive design parameters:

CUSUM control chart with adaptive reference parameter (Sparks, 2000):

$$C_t = \max\left[0,\ C_{t-1} + (x_t - \delta_t/2)/h(\delta_t)\right]$$

where $h(\delta_t)$ is function which maintains a constant control limit the shift magnitude δ_t is on-line updated using an EWMA-type equation

$$\delta_t = \max\left(wx_{t-1} + (1-w)\delta_{t-1},\ \delta_{\min}\right)$$

(suggested $\delta_{\min}$ is 0.5 for detecting smaller shifts, 1.0 for detecting shifts larger than 1.0)

3. Adaptive control charts

Examples of charts with adaptive design parameters:

EWMA adaptive procedure with adaptive smoothing parameter (Capizzi and Masarotto, 2003):

$$Z_t = \big(1 - w(e_t)\big)Z_{t-1} + w(e_t)x_t$$

where $e_t = x_t - Z_t$. For small values of e_t , $w(e_t)$ becomes relatively small, while for large e_t the value of $w(e_t)$ enlarges accordingly.

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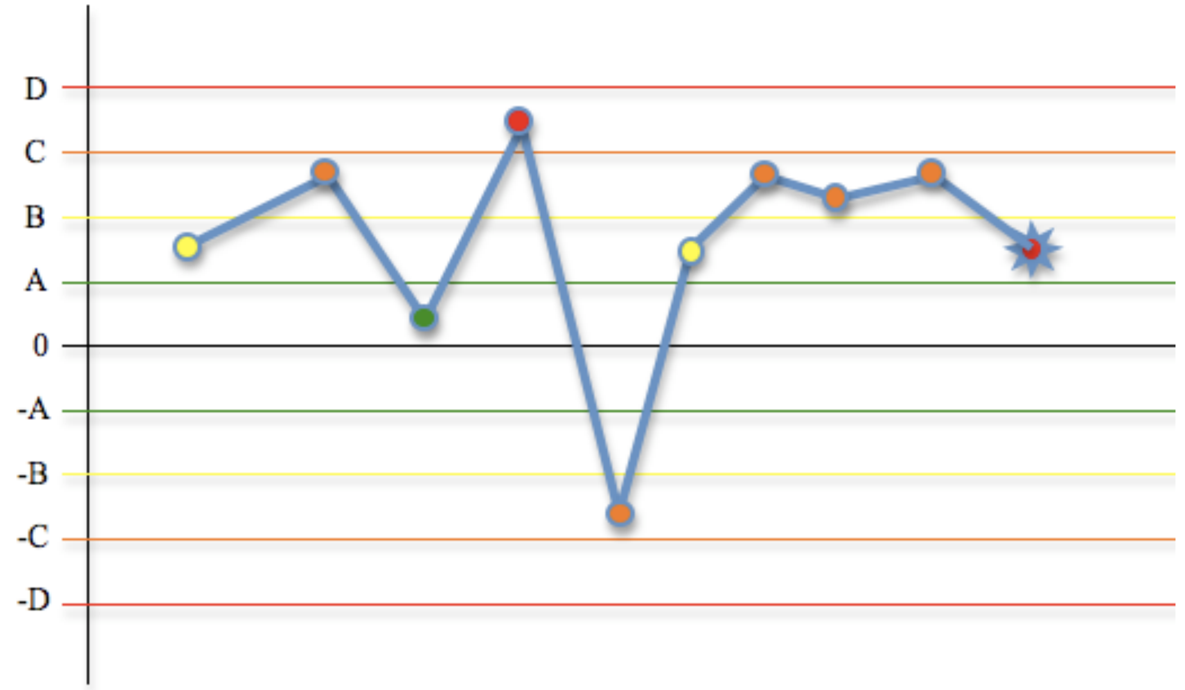
Examples of charts with adaptive design parameters:

Zone adaptive procedure with adaptive control limits:



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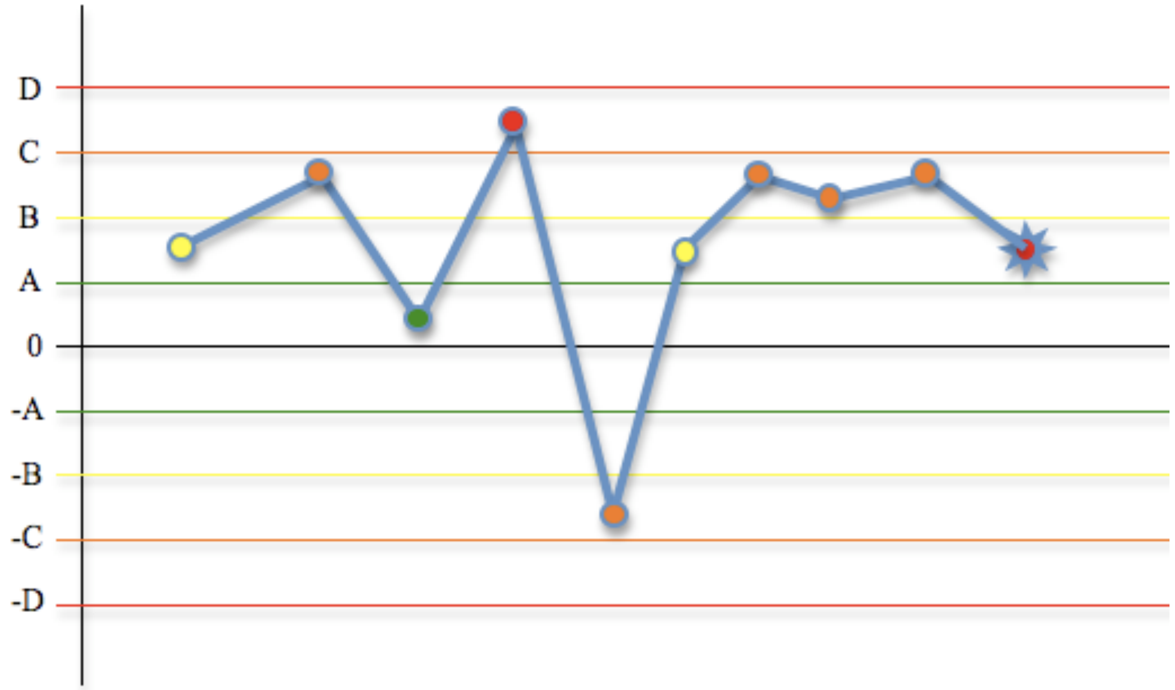

$$x \in Z \Rightarrow \text{NextLimit} = L_{n+1}(Z, L_n)$$

where

$$L_{n+1}(Z, L_n) = L_n - w(Z), \qquad \text{when } Z \neq \langle -A, A \rangle,$$

$$L_{n+1}(Z, L_n) = (D, -D) \qquad \text{when } Z = \langle -A, A \rangle,$$

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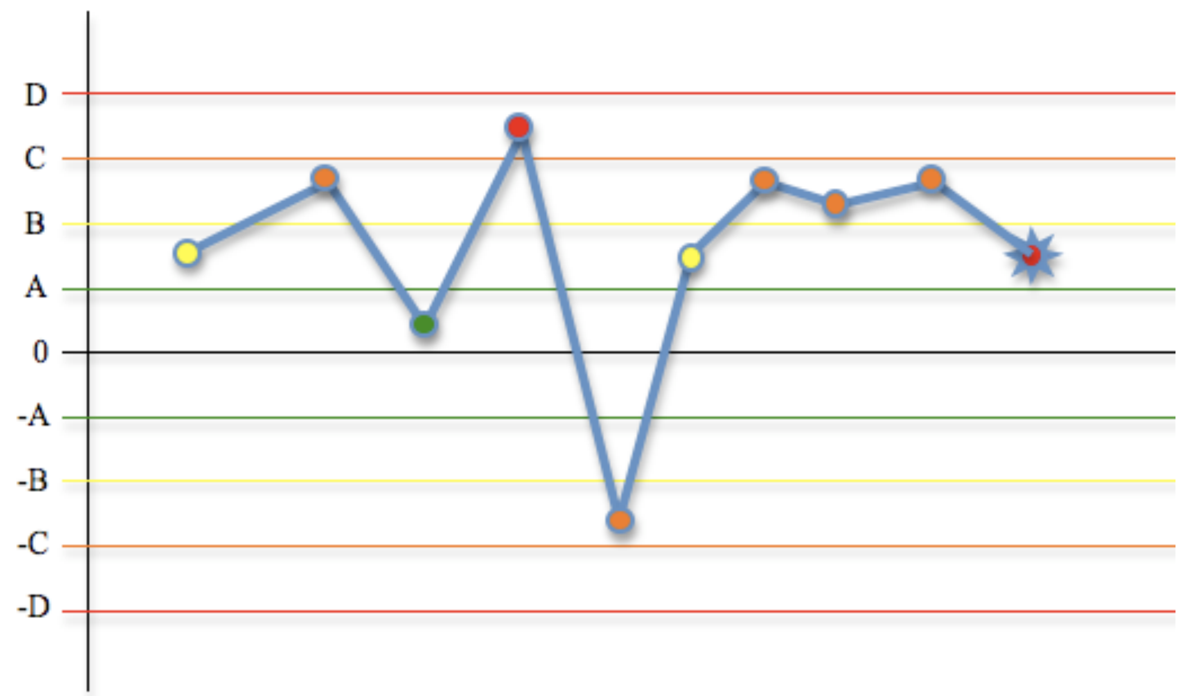

An example:

$$\tilde{x}_n \in (-D, -C\rangle \quad \Rightarrow \quad LCL_{n+1} = LCL_n + \sigma,\ UCL_n = D,$$

$$\tilde{x}_n \in (-C, -B\rangle \quad \Rightarrow \quad LCL_{n+1} = LCL_n + 1/2\sigma,\ UCL_n = D,$$

$$\tilde{x}_n \in (-B, -A\rangle \quad \Rightarrow \quad LCL_{n+1} = LCL_n + 1/4\sigma,\ UCL_n = D,$$

$$\tilde{x}_n \in (-A, A) \quad \Rightarrow \quad LCL_{n+1} = -D,\ UCL_n = D,$$

$$\tilde{x}_n \in \langle A, B) \quad \Rightarrow \quad LCL_{n+1} = -D\ UCL_n = UCL_n - 1/4\sigma,$$

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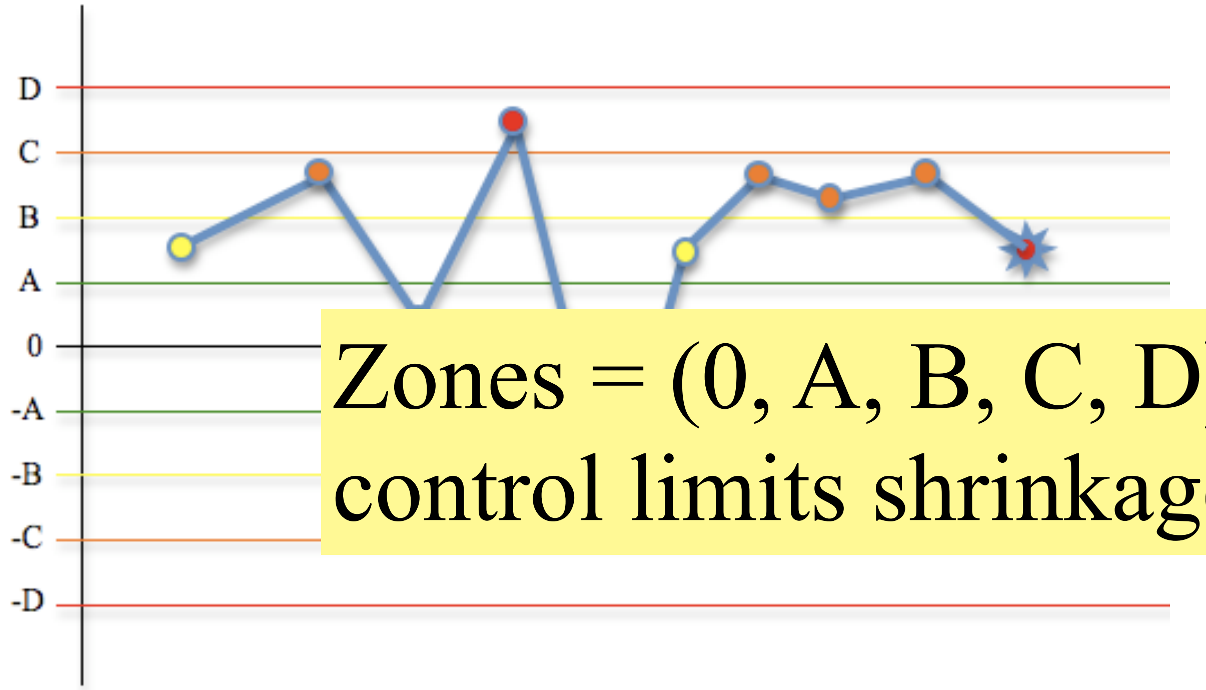

Zones = (0, A, B, C, D)
control limits shrinkage = (0, 1/4, 1/2, 1)

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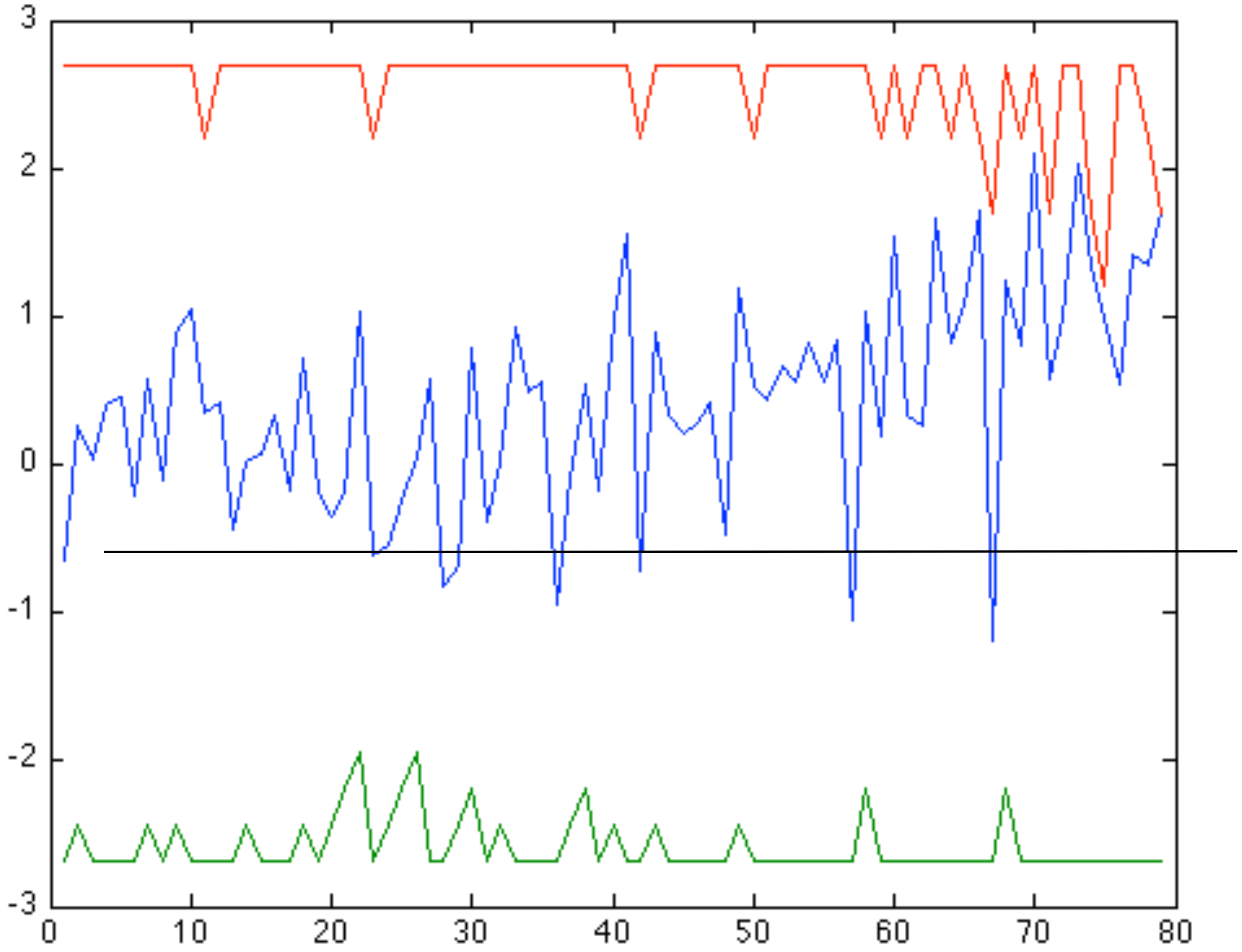

4. Robust adaptive (ROAD) control charts

CUSUM $\tilde{X}$

$$\tilde{C}^{+}_{n+1} = \max\left[0, \tilde{C}^{+}_{n} - (\mu_0 + \delta_0) + \tilde{X}_n\right]$$

$$\tilde{C}^{-}_{n+1} = \max\left[0, \tilde{C}^{-}_{n} + (\mu_0 + \delta_0) - \tilde{X}_n\right]$$

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Adaptive-CUSUM $\tilde{X}$ control chart $\delta_0 = 0.187$

zones (A, B, C) = (2.5, 3.0, 4.34)

CL shrinkage = (0 , 0.25, 0.5)

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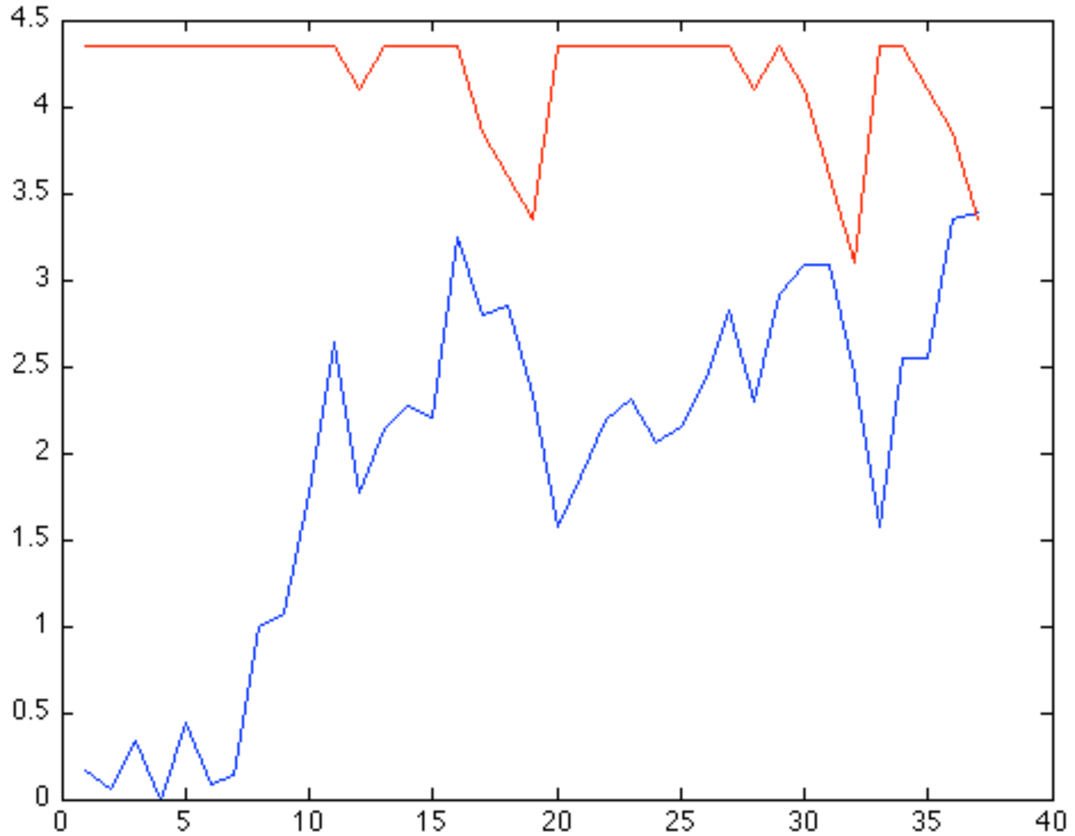

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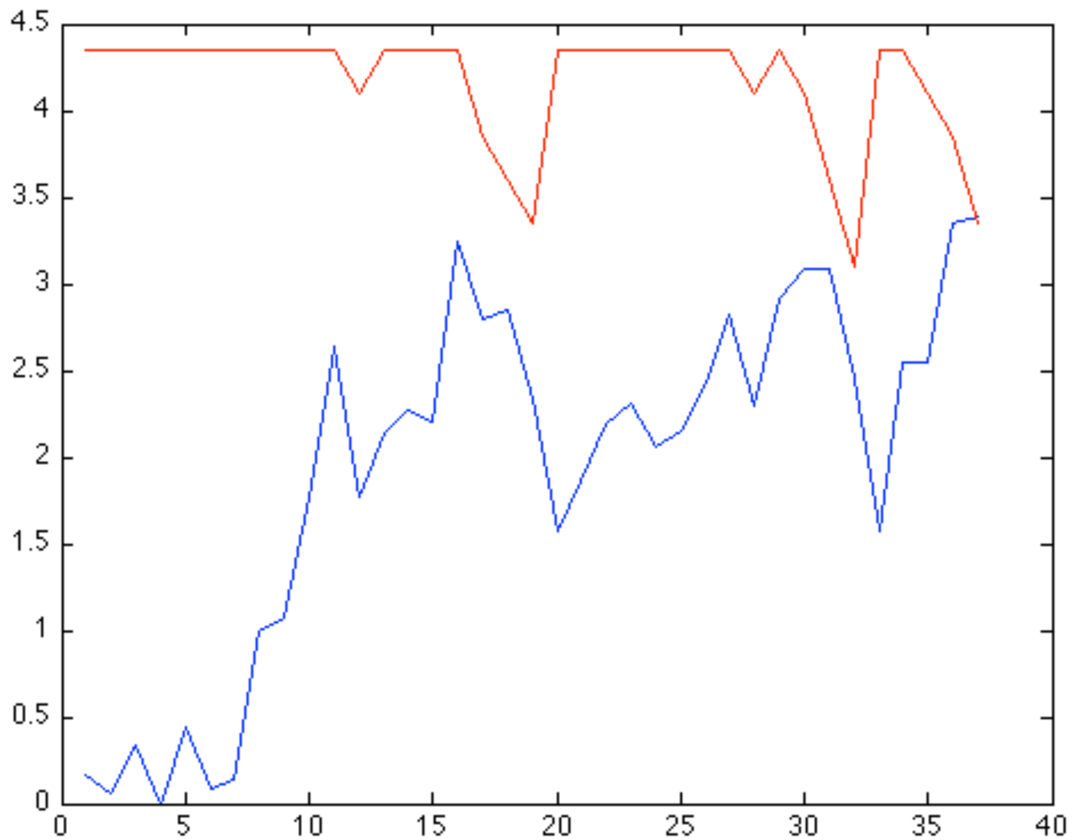

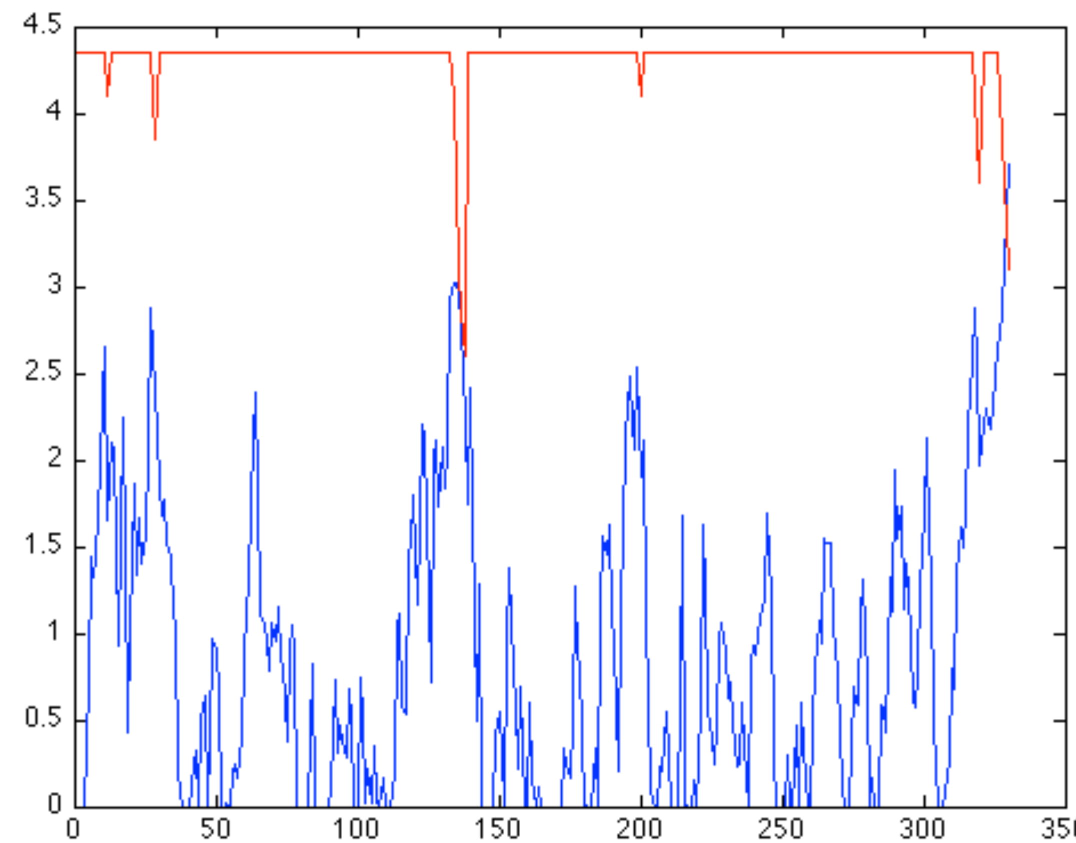

4. Robust adaptive (ROAD) control charts

Numerical results:

Data (simulation): data is from $N(\delta, 1)$, $\delta = 0,\ 0.1,\ 0.3,\ \ldots.$ (each sample size = 5). For contamination is used 6% of $N(\delta, 6.25)$.

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1) Shewhart $\tilde{X}$ control chart with UCL=3.128

2) EWMA $\tilde{X}$ control chart $\tilde{Z}_{n+1} = \gamma \tilde{X}_n + (1-\gamma)\tilde{Z}_n$

$$\gamma = 0.1, \quad L = 2.827$$

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4. Robust adaptive (ROAD) control charts

Numerical results: ARLs of different charts with data from N(δ,1)

Control Chart		shift δ						
		0.0	0.1	0.3	0.5	0.7	1.0	1.5
$\bar{X}$	Shewhart	500.2	405.3	128.1	41.5	16.3	5.0	**1.7**
	CUSUM	501.2	130.0	20.2	9.9	6.5	4.5	2.9
	EWMA	501.0	136.3	**19.2**	**8.5**	**5.2**	**3.4**	2.4
$\tilde{X}$	Shewhart	500.1	439.2	175.3	69.6	28.3	9.7	2.7
	CUSUM	502.0	151.4	26.2	13.1	8.7	5.7	3.8
	EWMA	499.2	165.4	24.4	10.3	6.4	4.0	2.7
$\tilde{X}$	Ad-CUSUM	504.1	**124.9**	24.5	11.7	7.9	5.4	3.7

Yang, L, Pai, S, Wang Y.R.: A novel CUSUM Median Control Chart. Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol. III, IMECS 2010, March 17 - 19, 2010, Hong Kong

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Numerical results: ARLs of charts with contaminated data

Control Chart		shift δ						
		0.0	0.1	0.3	0.5	0.7	1.0	1.5
$\bar{X}$	Shewhart	87.1	78.0	48.2	24.5	13.8	5.7	2.7
	CUSUM	265.4	97.4	19.2	9.9	6.6	4.4	2.9
	EWMA	186.1	85.2	17.1	8.0	5.0	3.4	2.4
$\tilde{X}$	Shewhart	264.5	236.8	126.6	48.6	24.1	9.8	4.0
	CUSUM	430.0	139.4	26.8	12.8	8.6	5.8	3.8
	EWMA	343.9	127.4	23.3	10.0	6.2	4.0	2.7
$\tilde{X}$	Ad-CUSUM	466.2	121.5	24.1	11.8	7.9	5.4	3.7

Data: mixture of 94% N(δ, 1) and 6% of N(δ, 6.25)

4. Robust adaptive (ROAD) control charts

Numerical results: Relative ARLs of charts with contaminated data

Control Chart		shift δ						
		0.0	0.1	0.3	0.5	0.7	1.0	1.5
$\bar{X}$	Shewhart	500	447.7	276.7	140.6	79.2	32.7	15.5
	CUSUM	500	183.5	36.2	18.7	12.4	8.3	5.46
	EWMA	500	228.9	45.9	21.5	13.4	9.1	6.45
$\tilde{X}$	Shewhart	500	446.1	238.5	91.6	45.4	18.5	7.54
	CUSUM	500	162.1	31.1	14.9	10.0	6.74	4.42
	EWMA	500	185.2	33.9	14.5	9.01	5.82	**3.93**
$\tilde{X}$	Ad-CUSUM	500	**130.3**	**25.9**	**12.7**	**8.47**	**5.79**	3.97

$$RARL_C(\delta) = k.ARL_C(\delta), \quad \text{where } k = \frac{ARL(0)}{ARL_C(0)}$$

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X	CUSUM	500	162.1	31.1	14.9	10.0	6.74	4.42
	EWMA	500	185.2	33.9	14.5	9.01	5.82	**3.93**
$\tilde{X}$	Ad-CUSUM	500	**130.3**	**25.9**	**12.7**	**8.47**	**5.79**	3.97

5. Conclusions:

In the case of contaminated data, improved CC using adaptive detection scheme is the best choice!

$$RARL_C(\delta) = k.ARL_C(\delta), \quad \text{where } k = \frac{ARL(0)}{ARL_C(0)}$$

Thank You for Your attention!

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